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Stress estimates from the length/width ratios of fractures

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Abstract—The hypothesis is advanced that, provided Young's modulus and Poisson's ratio of the rock are known, the length/width ratios of tension fractures can be used to estimate the tensile stress (assumed constant along the length of each fracture) at the time of fracture formation. The hypothesis is tested on a fissure swarm in a 10,000 year-old basaltic lava in Iceland. The length/width ratios of the fissures give the average tensile stress as of the order of a few MPa.

INTRODUCTION

IN THIS paper it is shown that provided two elastic moduli of the rock are known, the length/width ratios of tension fractures permit an estimate to be made of the tensile stress at the time of fracture formation.

The analysis is restricted to true tension fractures, that is extension fractures that form normal to a tensile least principal stress (compressive stress reckoned positive) (Secor 1965). The analysis may, however, be extended to hydraulic fractures, where σ_3 (the least principal stress) is compressive but the fluid pressure is sufficient to make the principal effective stress tensile. Then, however, the length/width ratio of the fracture gives an estimate of the principal effective stress, or the overpressure of the fluid, but not of the least principal stress itself. For instance, the length/width ratio of a dyke gives the overpressure of the magma at its time of formation, and from the overpressure calculated in this way, the depth of origin of the magma can be estimated (Gudmundsson 1983).

THEORY

In the analysis presented below some idealizations are made for the purposes of simplification and to make the problem tractable. (1) The rock is assumed to be a homogeneous, isotropic, elastic material. (2) The least principal stress, σ_3, is assumed to be constant along the length of the fracture, at its time of formation. (3) The fractures are assumed to result from an absolute tensile stress, that is σ_3 is assumed to have been negative when the fractures formed.

The first assumption is the usual one in this kind of an analysis and is generally justifiable (Jumikis 1979). The second assumption must be made in order to estimate the tensile stress from length/width ratios. If the least principal stress was variable along the length of the fracture, its variability would remain unknown. However, in a stress field due to remote regional forces, the tensile stress probably remains constant, or nearly so, along most of the (potential) length of the fracture provided the (potential) fracture lies completely within a single rock mass. By this I mean that the fracture originates in a rock that has essentially the same elastic properties along the length of the fracture. The third assumption is made to exclude from the study extension fractures that form as a result of local tensile stress concentrations near the ends of elliptical microscopic flaws (Griffith cracks) when the total value of σ_3 is compressive.

Consider an infinite plane with a straight crack subjected to a remote uniform tensile stress, $-p$ (Fig. 1).

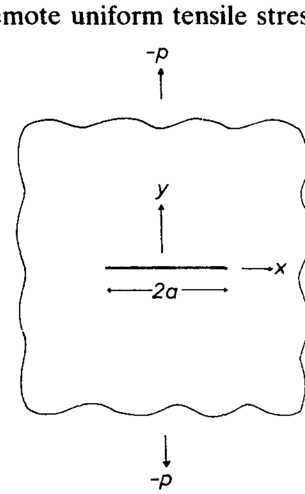

Fig. 1. A hypothetical crack of length $2a$ is subjected to a remote uniform tensile stress, $-p$. The crack is situated along the x-axis of the coordinate system and opens in the direction of the y-axis.

The crack is situated along the x-axis and occupies the segment $y = 0, |x| \leq a$. The shape of the crack is given by Parker (1981) as

$$v = -p[(\kappa + 1)/4\mu]\cdot(a^2 - x^2)^{1/2}, \qquad (1)$$

where v is the displacement of points on the crack surface in the y-direction, a is the half-length of the crack, and the factors κ and μ are given by:

$$\kappa = 3 - 4\nu, \quad \text{(plane strain)} \qquad (2)$$

$$\mu = E/[2(1 + \nu)], \qquad (3)$$

where ν is Poisson's ratio and E is Young's modulus.

Putting equations (2) and (3) into equation (1) gives:

$$v = -2p(1 - \nu^2)E^{-1} \cdot (a^2 - x^2)^{1/2}. \quad (4)$$

This equation can be rewritten as:

$$x^2/a^2 + v^2/b^2 = 1, \quad (5)$$

where b is given by:

$$b = -2pa(1 - \nu^2)E^{-1}. \quad (6)$$

Equation (5) is the equation of an ellipse, which shows that the remote uniform tensile stress, $-p$, opens the crack into an ellipse. At the centre of the ellipse, $x = 0$, v is at a maximum and equal to the half-width of the crack. If we now let $L = 2a$ be the length of the crack and $W = 2v$ be the width of the crack, then the maximum width, W_{max} (at $x = 0$), is [from equation (4)]

$$W_{max} = -2pL(1 - \nu^2)E^{-1}. \quad (7)$$

This equation can be solved for the length/width ratio,

$$L/W_{max} = E/[-2p(1 - \nu^2)], \quad (8)$$

or directly for the tensile stress:

$$-p = \frac{E}{2(1 - \nu^2)} \frac{W_{max}}{L}. \quad (9)$$

Equation (9) gives the tensile stress when the length/width (or width/length) ratios of the fractures are known, provided Young's modulus and Poisson's ratio are also known.

The dynamic value for Young's modulus, E_d, is given by Jaeger & Cook (1969) as:

$$E_d = \frac{V^2(1 + \nu)(1 - 2\nu)\rho}{(1 - \nu)}, \quad (10)$$

where V is the P-wave velocity, ν is Poisson's ratio and ρ is the density of the rock.

Equation (10) assumes a knowledge of Poisson's ratio. The dynamic Poisson's ratio can be calculated from the P- and S-wave velocities. However, because of uncertainties of the S-wave velocity, especially in the uppermost part of the crust, the dynamic Poisson's ratio is inaccurate. One is therefore compelled to estimate the in-situ Poisson's ratio from laboratory measurements on similar rock, bearing in mind that the in-situ values may be somewhat different from the laboratory values.

The use of the dynamic value of Young's modulus, when estimating tensile stress from length/width ratios of fractures, may not always be justifiable. The dynamic modulus is of the order of 10^{-3} s, but the static modulus, E_s, is 10^2–10^4 s. It is likely that some small fractures will develop as fast as the seismic waves, so the dynamic modulus should be used. But big fissures presumably take much longer time to form, so for those the static modulus should be used.

It is known that the static values of Young's modulus are generally significantly lower than the dynamic values (Jaeger & Cook 1969). However, the difference between the static and dynamic moduli depends on the rock in question and is variable. According to data given by Jaeger (1979, p. 42) and Jaeger & Cook (1969, p. 176) most rocks seem to have E_d/E_s ratio from about 1.4 to about 2.8. For our purpose it is sufficient to assume the $E_d/E_s =$ ratio to be 2.0.

To estimate the tensile stress from the length/width ratios of the fractures, we proceed as follows. First, calculate the length/width (or width/length) ratios; then estimate Poisson's ratio from the laboratory values for rocks similar to the host rock. From Poisson's ratio and the density of the host rock, calculate the dynamic Young's modulus and assume the value of the static modulus to be two times lower. Finally, use equation (9) to calculate the tensile stress.

APPLICATION

To illustrate the use of length/width ratios of fractures to estimate tensile stresses, I will consider the Vogar fissure swarm in southwestern Iceland (Fig. 2). This fissure swarm is described in detail by Gudmundsson (1980), but the length/width ratios of the fractures are taken from Gudmundsson (1978). Figure 3 is a detailed map of the Vogar swarm.

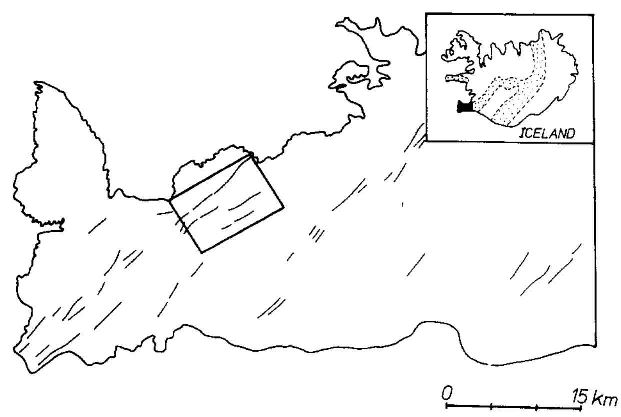

Fig. 2. Location of the Vogar fissure swarm on the Reykjanes Peninsula. Only a few of the biggest fractures are shown. On the small map of Iceland, the shaded area is the neovolcanic zone (the Reykjanes Peninsula being a part of it).

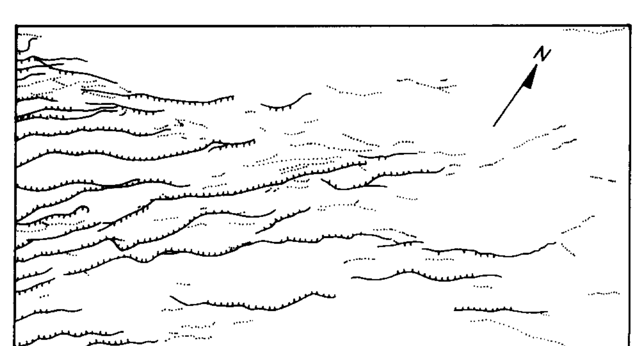

Fig. 3. A map of the Vogar fissure swarm. Only the fractures inside outcrop of the 10,000 year-old lava are shown (younger lavas are to the west of the area). 1 = fissure; 2 = normal fault. On the faults, each perpendicular line is a point where a significant throw has been measured. Only the fissures were used in the analysis given in this paper (modified from Gudmundsson 1980).

The Vogar fissure swarm lies almost completely within the outcrop of a single 10,000 year-old basaltic lava. Only data from the 68 extension fractures that lie completely inside the outcrop of the lava (i.e. do not dissect younger or older lavas) are included in the calculations presented below. These data were obtained from measurements on air photographs, but field work was also carried out to test the accuracy of the measurements. The accuracy of the data is 0.5 m for the width of the fractures and 10 m for the length.

The average length/width ratio of the fractures is 650. The P-wave velocity of Holocene basaltic lavas in Iceland is usually 1500–3000 m s^{-1} (Orkustofnun 1982). The velocity is variable inside individual lava flows, and as we are dealing with an area of about 80 km^2, it is clear that some average velocity must be used. We shall use the average of the above figures, that is 2250 m s^{-1}. The density of the uppermost 0–1 km of the crust inside the neovolcanic zone in Iceland is 2100–2500 kg m^{-3} (Palmason 1971), so 2300 kg m^{-3} is a reasonable figure for the density of the lava. Poisson's ratio of basalt is commonly about 0.25 (Jumikis 1979).

Taking the P-wave velocity to be 2250 m s^{-1}, the density to be 2300 kg m^{-3} and the Poisson's ratio to be 0.25, then equation (10) gives E_d as 9.7×10^9 Pa. Using the static modulus, E_s, and assuming that $E_s = 0.5E_d$, equation (9) gives the tensile stress ($-p$) as about 4 MPa. This indicates that, on average, the tensile stress that caused the fractures in the Vogar fissure swarm was of the order of a few MPa. This is somewhat less than the laboratory tensile strength of most basaltic rocks (Jumikis 1979) and thus indicates that the tensile strength of the lava was less than the usually determined laboratory values. This is to be expected because the lava contains numerous columnar joints that reduce the tensile strength.

DISCUSSION

There are several factors that need to be taken into account when applying the above method to big fissures like those in the Vogar swarm. First, fallen blocks from the fracture edges, because of weathering or subsequent movement on the fracture, may increase the apparent value of W_{max} with time. However, this factor is presumably well within the uncertainty in the measurements. Second, the fractures may have formed over a long period of time, so the L/W_{max} ratios measured today need not be the original ones. For the Vogar fissure swarm Gudmundsson (1980) concluded that fracture formation was not continuous, but that it was impossible to assert whether it was periodic or if all the fractures formed suddenly. Furthermore, it is not clear in what way the L/W_{max} ratios would change, should they change at all, if the fractures formed over a long period of time.

Third, it has been suggested that the fractures represent dykes in which the magma failed to reach the surface (Walker 1965). Even if this was so, it would not change this analysis. Using the principle of superposition (Parker 1981, p. 31, Fung 1965, p. 3) the tensile stress from equation (9) will be the same as before, but instead of being due to remote loading, the stress will now be related to the overpressure of the magma in the dyke below.

More important to the method than the factors above, is the accuracy of the values of the various 'constants' used in the calculations. As for Poisson's ratio and the density, the values of both are reasonably well known and the possible variation is too small to have significant effects on the results. As for Young's modulus, its value has significant effect on the stress estimate and is, unfortunately, known with less accuracy than either Poisson's ratio or the density. The dynamic Young's modulus depends mostly on the P-wave velocity [equation (10)], which, within a single layer, can be variable by a factor of two. From equation (10) this means that E_d can vary by a factor of four within the same layer. Usually, the static modulus, E_s, is about two times lower and its possible variation within a single layer will thus be by the same factor as the variation of E_d.

The variation of Young's modulus, either dynamic or static, by a factor of four is, however, probably an extreme case. In any one layer, it should be the highest Young's modulus that determines the final L/W_{max} ratio of a fracture. So, using the greatest P-wave velocity in that layer one should get the Young's modulus, dynamic or static, accurate within a factor of about two. As for the Vogar fissure swarm, the P-wave velocity in the surface lava is not known, and the value used in this paper is based on general measurements on surface lavas inside the neovolcanic zone. In such cases it is reasonable to use average values, but if the exact P-wave velocity was known one should use the maximum value. Nevertheless, the 'constants' for the Vogar swarm lava are certainly known with enough accuracy for the order of magnitude calculations above. Thus the results allow one to state that the tensile stress, at the time of formation of the Vogar fissure swarm, was of the order of a few MPa.

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